NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL NOTE

No. 1039

QUANTITATIVE TREATMENT OF THE CREEP OF METALS BY

DISLOCATION AND RATE-PROCESS THEORIES

By A. S. Nowick and E. S. Machlin

Aircraft Engine Research Laboratory
Cleveland, Ohio

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE NO. 1039

QUANTITATIVE TREATMENT OF THE CREEP OF METALS BY

DISLOCATION AND RATE-PROCESS THEORIES

By A. S. Nowick and E. S. Machlin

SUMMARY

(An equation for the steady-state rate of creep has been derived
by applying the theory of dislocations to the creep of pure metals.)
The form of this equation is in good agreement with empirical equa-
tions describing creep rates. The theory was also used to predict
the dependence of steady-state rate of creep on physical constants
of the material and good agreement was obtained with data in the
literature for pure annealed metals. (The rate of creep was found to
decrease with increasing modulus of rigidity) This fact suggests
that one of the requirements for a heat-resisting alloy is that its
matrix be a metal that has a high modulus of rigidity and therefore
a high modulus of elasticity.)

INTRODUCTION

The development of the gas turbine in combination with the jet
as a power plant for military aircraft has focused attention on the
need for heat-resisting alloys. (One of the criteria used to evaluate
heat-resisting alloys is creep resistance, that is, the resistance
to plastic deformation over a period of time. Current evaluation of
creep resistance is accomplished by creep tests in which creep curves
(elongation plotted against time) are obtained at constant stress and
temperature. A typical creep curve is shown in figure 1. The initial
stage, in which the slope of the curve or rate of creep is rapidly
decreasing, is commonly designated the primary or transient stage;
secondary or steady-state creep refers to the straight-line portion
of the creep curve. After a sufficient length of time, the rate of
creep increases in the region designated the tertiary stage. In
certain cases, the rate of creep continuously increases and cannot
be divided into these three stages (reference 1(a)). The steady-
state rate of creep is generally the criterion by which the creep
resistance of heat-resisting alloys is expressed. Because this rate
rapidly increases with temperature, creep becomes an important factor
in limiting safe-operating temperatures.

One of the first attempts to analyze steady-state creep by other than empirical methods was by Kanter (reference 2) in 1938. Further work of a fundamental nature was carried out by Kauzmann (reference 3) who applied Eyring's theory of liquid flow, which is a special application of the theory of rate processes, to the steady-state flow of metals. The problem of creep has also been attacked by the use of the theory of dislocations. A survey of results obtained thus far by this approach is reported by Seitz and Read (reference 4). Although these three theoretical treatments predict equations for the steady-state rate of creep that showed the correct dependence on stress and temperature, quantitative predictions cannot be made from them as to how physical and structural constants of materials affect the creep rate. These treatments therefore are not of great value in leading to the synthesis of new materials that will have better creep properties than those currently used for heat-resisting purposes.

In order to determine the dependence of steady-state creep rate on physical constants of materials, the investigation reported herein, which continues a fundamental approach to the problem of creep, was conducted at the NACA Cleveland laboratory in the early part of 1945. An equation for the steady-state rate of creep as a function of applied stress and temperature is derived by applying Eyring's theory of rate processes and the theory of dislocations to the problem. This investigation is part of a program being conducted at the Cleveland laboratory to evaluate the physical properties of heat-resisting alloys in terms of physical constants, which are either known or easily measurable, in order to minimize the number of tests as well as to make possible the prediction of new compositions and structures for better heat-resisting alloys than those currently used.

SYMBOLS

The following symbols are used in the theoretical analysis:

d_1 distance between atoms in slip direction

d_2 interplanar spacing (fig. 4(a))

E   modulus of elasticity

F(T)   function of temperature and material

ΔF_a free energy of activation per molecule

f   fraction whose value is about 1/2

G   modulus of rigidity at any temperature

G_0 modulus of rigidity at absolute zero

h   Planck's constant, 6.62×10^{-27} erg seconds

ΔH_a heat of activation per molecule

k   Boltzmann's constant, 1.38×10^{-16} ergs per molecule per °K

L   distance between imperfections in a single crystal (of the order of 1 micron)

N_d number of dislocations per unit area that intersect a plane normal to the slip plane and containing the slip direction

N_s number of sources of dislocations per unit volume

p   probability of occurrence of oscillation in crystallographic direction under consideration

P^+ ratio of sources of dislocations at which a positive activated complex has formed to total number of right-hand sources of dislocations; that is, probability of formation of a positive activated complex at right-hand source

P^- probability of formation of a negative activated complex at right-hand source

$P' = -k \log_e p^5$

q   stress concentration factor

r   rate of reaction per unit concentration of reactants

R_g rate of generation (number of dislocations generated/sec for single generating source)

R_g^+ rate of generation of positive dislocations at right-hand source

R_g^- rate of generation of negative dislocations at right-hand source

ΔS_a entropy of activation per molecule

t   average time for activated complex to pass into product form

T   absolute temperature, °K

u   tensile creep rate at steady-state conditions

u'   shear rate

v   average velocity of motion of dislocations

V   volume associated with one atom

W   elastic energy

W_t energy to be attained by thermal motion

x   ratio of d_1 to d_2

α temperature coefficient of modulus of rigidity

δ displacement shown in figure 5(a)

μ Poisson's ratio

σ externally applied tensile stress

τ shear stress

τ_b back stress

τ_e externally applied shear stress

τ_s average shear stress at sources of dislocations

τ_t shear stress to be attained by thermal motions

τ_0 critical local shear stress for generation of dislocation

<center>THEORY</center>

The features of the theory of rate processes and the theory of dislocations required for the development of the creep equation are first reviewed; the derivation of the equation for the steady-state rate of creep then follows.

<center>Eyring's Theory of Rate Processes</center>

The theory of rate processes (reference 5), as developed by Eyring and others, considers a reaction or any rate process to be the result of the crossing of a potential-energy barrier by molecules

whose energies have exceeded a certain minimum value. The reaction
consists of the formation of an "activated complex" capable of cross-
ing the barrier followed by the passage of this complex over the
barrier. (See fig. 2.) The most important assumption is that the
initial reactants and activated complexes are always in equilibrium.
Application of thermodynamic and statistical mechanical considera-
tions shows that the number of activated complexes passing over the
barrier per second (or rate of reaction) per unit concentration of
reactants is given by

$$r = \frac{kT}{h}e^{-\Delta F_a/kT} \tag{1}$$

The term kT/h can be regarded as the effective frequency at
which activated complexes cross over the barrier; the exponential
factor represents the probability of formation of an activated
complex. In the calculation of this probability term, the contri-
bution due to the translational degree of freedom along the "reaction
coordinate" (the most favorable reaction path on the potential-energy
surface) has been disregarded because it is included in the factor
kT/h. The free energy of activation ΔF_a is interpreted as an
ordinary free-energy term and can be expressed by

$$\Delta F_a = \Delta H_a - T\Delta S_a \tag{2}$$

where the heat of activation per molecule ΔH_a is the height of the
potential-energy barrier in figure 2.

General Features of Dislocation Theory in Relation to Slip

The theory of dislocations arose in relation to the mechanism
of slip in single crystals of metal. Slip is the plastic deformation
that occurs in an arbitrary short period of time when a metal is
subjected to stress. This deformation is characterized by the dis-
placement of one part of the crystal lattice relative to another
along a particular crystallographic plane and in a definite crystal-
lographic direction. These slip planes and slip directions are
detected by the occurrence of slip bands, step-like discontinuities
on the surface of single crystals that have been subjected to stress.
The spacings of the bands are generally of the order of 1 micron
(reference 1(b)). For any given crystallographic plane, there is a
more or less critical value of the component of the applied stress
above which the rate of plastic deformation rapidly increases. This
value is called the critical shearing stress. Early attempts to
explain the mechanism of slip based upon the assumption that the
process consists in the sliding of parallel planes over one another
lead to theoretically determined values of the critical shearing

stresses, which were of the order of 1000 times too large. It there-
fore became evident that the true nature of slip involves deviations
from a perfect lattice.

Of several mechanisms suggested the one that has been most
successful in explaining the slip process is the theory of dislo-
cations. This theory, which was advanced by G. I. Taylor (refer-
ence 6), Orowan (reference 7), and Polanyi (reference 8), proposes
that local deviations from a perfect lattice called dislocations
exist in single crystals and that movement of dislocations through
the stressed crystal produces slip. A dislocation consists in a
stable arrangement of atoms such that, in a region of a few atomic
distances, n + 1 atoms in the slip direction face n atoms across
the slip plane. The point at which the atoms are one-half an atomic
spacing "out of step" is called the center of the dislocation. The
extension of the dislocation in the slip plane and normal to the
slip direction is called the length of the dislocation.

A simplified model of a dislocation in which two neighboring
planes are shown in cross section is presented in figure 3; the
plane of the figure is normal to the length of the dislocation. A
small shear stress applied as shown in figure 3(a) will cause a
displacement of the center of the dislocation by one atomic distance
(fig. 3(b)). The dislocation finally passes completely out of the
crystal and a perfect lattice is restored, differing from the orig-
inal lattice in that one plane has now been translated relative to
the other by one atomic distance. In the process just described,
the entire linear dislocation must move as a unit.

Barrett (reference 9) presents a sketch taken from an earlier
work by Taylor (reference 6) showing the two types of dislocation
that can exist (fig. 4). If, in the neighborhood of the center of
the dislocation, more atoms are above the slip plane, a positive
dislocation occurs and, if more atoms are below, a negative dislo-
cation occurs. For the same shear stress, positive and negative
dislocations move in opposite directions but the net result of the
passage through the crystal is the same, a translation of one part
of the lattice with respect to the other by one atomic distance
(fig. 4).

Inasmuch as the passing of a dislocation out of the crystal
lattice results in the loss of the dislocation, new dislocations
must be generated in order for continuous plastic deformation to be
possible. A dynamic theory of dislocations is therefore necessary
and was developed by Orowan (reference 7) and extended by others,
including Seitz and Read (reference 4). Because of the great energy
required, the process of generation generally takes place at regions
of high stress concentration resulting from imperfections in the

single crystals. The existence of such imperfections as a result of accidents of growth during the formation of single crystals is generally accepted but the nature of the flaws that exist is still a matter of controversy. For example, the theory that single crystals are made up of a mosaic block structure is discussed in reference 4. X-ray evidence indicates that the average spacing of the imperfection is of the order of 1 micron. The general dynamic picture obtained is as follows: Dislocations are generated under shear stress at the regions of high stress concentration, which will be called sources of dislocations. A source of dislocations will be called a right-hand source if it generates dislocations to the right and a left-hand source if it generates dislocations to the left. The dislocations as they are generated are only a few atoms long because of the extremely low probability of generating a full-length dislocation (one whose length is of the order of the spacing between imperfections) (reference 4). As the dislocations move through the crystal under shear stress, they increase in length and move in the slip plane and slip direction until they become stuck at an imperfection or in the vicinity of other dislocations.

Further properties of dislocations and their use to explain the qualitative phenomena of plastic deformation are summarized in a series of four papers by Seitz and Read (reference 4).

Derivation of an Equation for Creep Rate

In the analysis of the creep process, it will be assumed as in reference 4 that creep as well as slip takes place by the motion of dislocations. The dynamic model previously described will be used. The procedure is as follows: An equation will be derived for the steady-state rate of creep, which will show that the rate of generation of dislocations is the primary factor that determines the variation of creep properties among materials. The rate of generation will then be calculated in terms of rate-process theory as a frequency factor multiplied by the probability that a source of generation will attain the energy required to generate a dislocation.

For a given block of volume L^3, if it is assumed that no "annihilation" (union of pairs of dislocations of opposite sign) takes place and that steady-state creep is being dealt with, the number of dislocations generated per second within the block must equal the number leaving the block per second. The number of dislocations leaving the block per second is the number of dislocations in an area vL so

$$R_g N_s L^3 = N_d vL \qquad (3)$$

Inasmuch as the displacement produced when a dislocation passes out of the block is d_1, the shear rate u' is given by

$$u' = \frac{1}{L}(N_d vL)d_1 = N_d v d_1 \tag{4}$$

The occurrence of slip bands at distances apart of the order of 1 micron indicates an average of one source of dislocations for each imperfection; that is

$$N_s = 1/L^3 \tag{5}$$

When equations (3), (4), and (5) are combined, the expression for shear rate becomes

$$u' = \frac{d_1}{L} R_g \tag{6a}$$

Up to this point, only the creep of single crystals has been considered. In a consideration of polycrystalline metals, it would appear that two types of creep may take place: creep occurring within the individual grains or single crystals, and creep resulting from intergranular motion. As noted in reference 4, present data on the effect of grain size on creep leaves doubt as to the possibility of the occurrence of intergranular creep. For example, the work reported in reference 10 on copper of various grain sizes shows no detectable difference in creep strength other than that resulting from oxidation. Unfortunately, this work was carried out only at a single temperature. Creep in polycrystalline metals is therefore assumed to be predominately the result of deformation occurring within the individual grains by the motion of dislocations. If the orientation of the grains is such that slip occurs in the plane of maximum shear stress, then, as in reference 3, the strain rate u' can be corrected to the tensile creep rate u by an additional factor of about two-thirds. Such a factor would be of no consequence here because d_1/L is known only in order of magnitude; accurate values cannot be given until more is known about the nature of the crystal imperfections. Equation (6a) can therefore be written approximately as

$$u = \frac{d_1}{L} R_g \tag{6b}$$

The rate of generation R_g will now be calculated. A specific model for the mechanism of generation of a dislocation must be set up and certain simplifying assumptions made. The approximations that will be made are similar to those used by Orowan (reference 7) but the treatment presented here is based upon a more definite model than he used. Furthermore, considerations in reference 7 were not applied to steady-state creep.

The generated dislocation will be taken to be 1 atom long.
(The length of a dislocation is its extension in the slip plane and
perpendicular to the slip direction.) The process of generation
will involve displacements of atoms lying along the slip direction
by large fractions of an atomic distance. A simple model for the
generation of a positive dislocation at a right-hand source is pre-
sented in figures 5(a) and 5(b). If atoms 1 and 2 are simultaneously
displaced in the directions shown in figure 5(a) by large fractions
of the lattice spacing and atoms 3, 4, and 5 are moving in the slip
direction as shown, the generation of a dislocation becomes possible.
Before a dislocation will form, however, a rearrangement of the
neighboring atoms must take place in order to achieve a stable con-
figuration. Figure 5(b) shows the rearranged condition. Comparison
of figure 5(b) with the positive dislocation in figure 4(b) shows
that the two are essentially the same. The new positions of atoms
5 and 6 after rearrangement were found by using the approximate
method used by Taylor (reference 6) in his discussion of the motion
of a dislocation under stress. The contributions to the potential
field in row B by the neighboring rows A and C are taken to be sinu-
soidal with periods equal to the spacing of atoms 1 and 3 in row A
and atoms 2 and 4 in row C. Figure 5(c) shows that addition of the
two sinusoidal potential functions gives a resultant potential
function having four minima. Atoms 5, 6, and 7 can be expected to
fall into the deepest of these minima if atom 5 had begun by oscil-
lating to the left as in figure 5(a).

The displacement δ of atoms 1 and 2 will be some fraction f
of the atomic separation d_1 in the slip direction. A value of f
of somewhat less than one-half makes it possible to form a stable
configuration with a minimum activation energy.

In the calculation of R_g, it will be necessary to use Eyring's
general theory of rate processes. A right-hand source, under a
shear stress τ_g having the direction shown in figure 5, will be
considered. The "reactant" is the perfect lattice at the region of
high stress concentration. The activated complex for a positive
dislocation, which for brevity will be called a positive activated
complex, can be taken as the configuration shown in figure 5(a) and
the product as the dislocation after atomic rearrangement has taken
place (fig. 5(b)). If the fundamental assumption of rate-process
theory that equilibrium exists between reactant and activated complex
is made, the considerations involved in the general theory of rate
processes (reference 5) can be applied to the generation of dislo-
cations. From the theory of rate processes, the rate of generation
of positive dislocations R_g^+ can be expressed as

$$R_g^+ = \frac{1}{t} P^+ \qquad\qquad (7)$$

The discussion in reference 5 (p. 189) shows that, if the displacement involved in the transition from the activated complex to the final state is on the order of 10^{-8} centimeters, the factor $1/t$ is approximately equal to kT/h. In the formation of a dislocation, this transition corresponds to passing from the configuration of figure 5(a) to that of figure 5(b), which clearly involves a displacement of this order of magnitude. Hence, R_g^+ is given approximately by

$$R_g^+ = \frac{kT}{h} P^+ \qquad (8a)$$

Similarly

$$R_g^- = \frac{kT}{h} P^- \qquad (8b)$$

The factors P^+ and P^- will now be calculated. In order to simplify the calculation, the theory of elasticity is assumed to be applicable on an atomic scale. The generation of a dislocation will be treated as a "local shear," which takes place by means of thermal oscillations. The energy involved in this process will be calculated by the theory of elasticity. Furthermore, Hooke's law will be assumed valid for large strains. The use of these assumptions is necessary because the forces between individual atoms have not been evaluated in terms of physical constants but it should be remembered that the approximations may be serious enough to lead to large errors in the results.

The thermal atomic oscillations in figure 5(a) will be regarded as resulting in a local shearing of atoms 1, 6, and 2 producing a strain of δ/d_2. Inasmuch as the elastic energy per unit volume for a shear stress τ is $\tau^2/2G$ and the volume of the region of shear is $2V$, the elastic energy involved is

$$W = \frac{V\tau^2}{G} \qquad (9)$$

The result will be applied to the generation of a dislocation as follows: Let τ_0 be the local shear stress attained by atoms 1, 6, and 2 in figure 5(a). If the displacement δ is fd_1, application of Hooke's law to the displacement yields

$$\tau_0 = \frac{Gfd_1}{d_2} \cong Gxf \qquad (10)$$

The stress to be produced by thermal motion is

$$\tau_t = \tau_0 - \tau_s$$

if it is assumed that the local shear stress to be attained by
atoms 1, 6, and 2 in figure 5(a) must be τ_0, independent of applied
stress. The stress τ_s depends on the externally applied stress and
is in the same direction as τ_0. From equation (9) the energy to be
attained thermally W_t therefore is

$$W_t = \frac{V}{G} (\tau_0 - \tau_s)^2 \qquad (11)$$

Inasmuch as τ_s can be expected to be only a small fraction of τ_0
for rigid materials, the term in τ_s^2 may be neglected relative to
the other terms in the binomial expansion of $(\tau_0 - \tau_s)^2$. The
approximation obtained is therefore

$$W_t = VGx^2 f^2 - 2Vxf\tau_s \qquad (12)$$

where equation (10) has been used.

In order to evaluate τ_s, work-hardening must be considered.
A crystal in the steady-state range of creep is generally work-
hardened because deformation has taken place. From the nature of a
dislocation it is clear that a stress field exists about it. By
making use of this stress field, Taylor (reference 6) has explained
work-hardening on the basis of a "lattice" of stuck dislocations.
Taylor's explanation is a static one and is applied only to deter-
mining the form of the stress-strain curve. In terms of a dynamic
approach, the existence of a stress field about a dislocation leads
to the concept suggested by Kochendörfer (reference 11) that the
lattice of stuck dislocations creates an inner stress field whose
direction at the point of generation is opposite to the externally
applied shear stress τ_e in the slip plane. The value of the inner
stress field at the point of generation will be called the back
stress τ_b. The dynamic concept of hardening therefore consists in
the formation of a lattice of stuck dislocations, which results in
a back stress at the point of generation. This back stress lowers
the effective shear stress at these points and as a result the rate
of generation, hence the creep rate, decreases. These considerations
lead to the result that

$$\tau_s = q(\tau_e - \tau_b) \qquad (13)$$

Inasmuch as τ_b is always less than τ_e, the directions of τ_e and
τ_s are the same.

In a large assembly of systems, which are in thermal equilibrium
with each other, the fraction of them that will have energies greater
than some arbitrary value ϵ is given by $e^{-\epsilon/kT}$. In addition to

the energy requirement, the occurrence of the configuration of figure 5(a) involves a limitation on the direction of atoms 1, 2, 3, 4, and 5 to a particular crystallographic direction within a very small solid angle. If the probability of oscillation in this direction is p, the total probability of the configuration of figure 5(a), that is, the probability that a right-hand source will form a positive activated complex is given by

$$P^+ = p^5 e^{-W_t/kT} = p^5 e^{-(VGx^2 f^2 - 2Vxf\tau_s)/kT}$$ (14a)

Similarly the probability of forming a negative activated complex (the configuration of fig. 5(d)) at a positive source is

$$P^- = p^5 e^{-(VGx^2 f^2 + 2Vxf\tau_s)/kT}$$ (14b)

inasmuch as τ_s is opposite to τ_0 in this case. The applied stress τ_e therefore makes the generation of a positive dislocation more probable than the generation of a negative dislocation at a right-hand source.

If a negative dislocation is generated in the region under discussion, it would move to the left and out of the crystal without producing deformation if it were the only dislocation present. The last positive dislocation generated from this source, however, became stuck after moving through part of the crystal lattice; that is, all forces upon it were balanced. The generation of a negative dislocation at the point being considered will upset this equilibrium and attract the previous positive dislocation. As the two dislocations approach each other, the stress that each exerts on the other increases (from the equation for the stress field about a dislocation, reference 6) and a union takes place resulting in the annihilation of the two dislocations and restoration of a perfect lattice. The net rate of generation R_g, which is the rate of generation of positive dislocations that will eventually produce plastic deformation, is $R_g^+ - R_g^-$ or, from equations (8a), (8b), (13), (14a), and (14b)

$$R_g = \frac{2kT}{h} e^{-(VGx^2 f^2 + P'T)/kT} \sinh [2qVxf(\tau_e - \tau_b)/kT]$$ (15)

where

$$P' = - k \log_e p^5$$ (16)

The equation for the net rate of generation of negative dislocations at a negative source is clearly identical with equation (15). Equation (15) therefore gives the expression for the factor R_g of equation (6b).

According to this derivation, it is now clear that the generation of a dislocation 2 atoms in length involves the simultaneous oscillation of twice as many atoms with sufficiently great energies. Thus, the probability of such a generation taking place can be neglected as compared with the probability of generating a 1-atom dislocation.

The hyperbolic sine term in equation (15) arose from the lowering of the potential-energy barrier in the direction of the applied stress. The heat of activation per molecule ΔH_a (the height of the barrier for zero applied stress) and the entropy of activation per molecule ΔS_a are (by comparison with equations (1) and (2))

$$\Delta H_a = VGx^2f^2 \tag{17a}$$

$$\Delta S_a = -P' \tag{17b}$$

P' is always positive because p is less than 1 (equation (16)), therefore ΔS_a is negative.

When the general expression for R_g given by equation (15) is combined with the creep equation (6b), the complete equation for the steady-state rate of creep becomes

$$u = \frac{2d_1}{L} \frac{kT}{h} e^{-(VGx^2f^2 + P'T)/kT} \sinh \left[qVxf(\sigma - 2\tau_b)/kT \right] \tag{18}$$

where the applied tensile stress σ has been substituted for $2\tau_e$ assuming that slip in the individual grains occurs in the plane of maximum shear stress. For high values of applied tensile stress σ the hyperbolic sine function, to a good approximation, can be replaced by an exponential and equation (18) becomes, after taking logarithms of both sides

$$\log_e u = \log_e \left(\frac{d_1}{L} \frac{kT}{h} \right) - \left(\frac{VGx^2f^2 + P'T}{kT} \right) + \frac{qVxf}{kT} (\sigma - 2\tau_b) \tag{19}$$

Equation (18), or its approximation equation (19), is the form of the theoretical equation to be used for the steady-state creep rate of pure polycrystalline materials as a function of stress and temperature.

COMPARISON OF THEORETICAL AND EXPERIMENTAL CREEP EQUATIONS

The evaluation of the back stress τ_b as a function of temperature and applied stress is difficult and has not yet been accomplished. A great deal may be learned, however, from a comparison of the theoretical equation (19) with empirical equations. It has been empirically

determined that a hyperbolic sine function represents the dependence of creep rate on applied tensile stress σ for many polycrystalline materials (reference 12) in agreement with equation (18). At high values of stress, the hyperbolic sine function reduces to an exponential. In reference 13 creep data are presented, which show that a relation of the form

$$\log_e \left(\frac{u}{T}\right) = -\frac{a}{T} - b + \gamma\sigma \qquad (20)$$

represents the experimental data in the range of high stresses, where a and b are constants for a given material and γ is a function of temperature but not of stress. If the notation is so changed as to make this equation similar to equation (19), equation (20) can be rewritten

$$\log_e u = \log_e \left(\frac{d_1}{L}\frac{kT}{h}\right) - \frac{A + BT}{kT} + \frac{\beta\sigma}{kT} \qquad (21)$$

where

$A = ka$

$B = kb + k \log_e \left(\frac{d_1 k}{Lh}\right)$

$\beta = \gamma kT$

Comparison of the forms of equations (19) and (21) show good agreement. The simultaneous validity of the two equations requires that τ_b be a linear function of stress. In addition, the condition $\tau_b = 0$ for $\sigma = 0$ leads to the following expression for τ_b

$$\tau_b = F(T)\sigma \qquad (22)$$

Equation (21) shows that three factors completely determine the steady-state rate of creep: the constants A and B and the temperature function β. A comparison of equation (21) with equations (19) and (22) shows that these factors are expressed in terms of physical constants of the material as

$$A = VG_0 \ f^2 x^2 \qquad (23)$$

$$B = P' - Vf^2 x^2 G_0 \alpha \qquad (24)$$

$$\beta = qVxf(1 - 2F(T)) \qquad (25)$$

where the modulus of rigidity G has been taken in the approximate form

$$G = G_0(1 - \alpha T) \qquad (26)$$

COMPUTATIONS AND PRECISION

Creep data were obtained from literature for a number of poly-crystalline "pure" metals that had been annealed (references 13 to 20). (Although the references do not generally state the impurity content, the metals will be considered pure as compared with alloys.) The reproducibility of the creep data from the various sources is not known. From these data, curves of log creep rate against stress were plotted and straight lines were obtained. In terms of equation (21), the coefficient β/kT divided by 2.3 is the slope of this straight line and the intercept is equal to

$$\log\left(\frac{d_1}{L}\frac{kT}{h}\right) - \frac{(A + BT)}{2.3\ kT}.$$

The values of β and H (defined as $A + BT$) were then determined from the slopes and the intercepts, respectively. Finally, A and B were separately determined from a linear plot of H against T where d_1/L was taken as 10^{-4} for all materials.

The errors involved in the determination of A, B, and β depend in part on the number of stresses for which data were available at each temperature and on the number of temperatures at which observations were made. The ranges in temperatures and stresses used will also affect the error. Generally, the errors for A, B, and β were estimated to be about $\pm 5 \times 10^{-13}$ ergs, $\pm 8 \times 10^{-16}$ ergs per $^\circ K$, and ± 30 percent, respectively.

For a large number of pure polycrystalline metals, it was found that β varied with temperature. The function

$$\beta = C_1 e^{C_2 T}$$

approximates the temperature dependence of β. This function was determined over comparatively short ranges of temperature (200° to 300° K) and therefore may not apply over a large temperature range.

Values of the modulus of rigidity G were obtained from references 21 and 22. These references showed that variations in the value of G for a given material are about ± 5 percent. Room-temperature values were used as approximations of the absolute-zero values of G.

RESULTS AND DISCUSSION

Factor A

Equation (23) gives the theoretical expression

$$VG_0 f^2 x^2$$

for the factor A of the empirical equation (21). In figure 6,
A/V is plotted as a function of G_0 x^2. The line drawn is the best
straight line through the origin and represents the data within the
experimental error of A. From the previous theoretical discussion,
the slope of this line is f^2 and is equal to 0.25 or less. Because
the activation energy predicted on the basis of the validity of
Hooke's law will be greater than the actual value, it can be expected
that the theoretical prediction will be too high. The experimental
slope from the straight line of figure 6 is 0.14.

The agreement between theoretical prediction and experimental
results is encouraging when the assumptions made and the variety of
sources of data are considered.

Factor B

When equations (19) and (22) were compared with the empirical
equation (21), B was found to be equal to

$$P' - Vx^2 f^2 G_0 \alpha$$

where α is determined from the approximate relation

$$G = G_0(1 - \alpha T)$$

The order of magnitude of the term $Vx^2 f^2 G_0 \alpha$, which is 3×10^{-16} ergs
per $°K$, is within the estimated error of B, which is $\pm 8 \times 10^{-16}$ ergs
per $°K$; this term can therefore be neglected in comparing the
B values for different materials. The values of B for different
metals are presented in the following table:

Material	B ($ergs/°K$)	Reference
Aluminum	40×10^{-16}	13
Copper	45	14, 15
Iron	39	17
Lead	54	20
Nickel	33	16
Platinum	42	13
Silver	36	13
Tin	47	19

A correlation exists between the scatter of data presented in
figure 6 and the scatter of B values in this table. The straight
line in figure 6 falls between the experimental points for aluminum

and iron. The table shows that aluminum and iron have B values
equal to 40 × 10^{-16} and 39 × 10^{-16} ergs per °K, respectively. The
points above the line in figure 6 are all related to metals that have
B values less than 40 × 10^{-16} ergs per °K and the points below the
line correspond to B values greater than 40 × 10^{-16} ergs per °K.

The correlation between the direction of scatter of data for A
and B is satisfactorily explained by considering the method used to
obtain experimental values of A and B. For each material, values
of H (defined as A + BT) were plotted against T and the values
of A and B were obtained from the intercept and slope, respec-
tively, of the best straight line drawn through the points. For a
given material, the value of the intercept A will increase as the
slope B of the line drawn through the experimental points is
decreased. In view of this explanation, the scatter of data for A
about the theoretical line appears to result from experimental error
alone. It can also be concluded that the correct values of B fall
about their mean value much more closely than those in the table.

When the term containing α is neglected, B becomes equal to
P', which from equation (17b) is the negative of the entropy of
activation ΔS_a. The high negative value of the entropy of activa-
tion in the creep process has been pointed out in references 3 and 13
but satisfactory quantitative explanations of the magnitude of this
factor could not be given.

On the basis of the present treatment, the restriction of direc-
tion of 5 atoms participating in the generation of an activated com-
plex accounts for the large negative entropy of activation. A good
approximation of the value of the probability of oscillation p of
an atom in the slip direction can be obtained by substituting values
of B for P' in equation (16). A value of about 3 × 10^{-3} was
obtained for p when P' was equal to 40 × 10^{-16} ergs per °K.

Factor β

The theoretical expression for the factor β is given by

$$\beta = qVxf(1 - 2F(T))$$

The term $F(T)$ in this equation is related to the back stress τ_b
(equation (22)), which resulted from the inner stress field about a
dislocation. This stress field is related to the modulus of rigid-
ity G (reference 6). Therefore, β at any given temperature was
expected to be a function of G. Figure 7 shows that at room temper-
ature β varies as G^{-n} where n is approximately 3.

In order to determine whether the temperature dependence of G could alone account for the temperature dependence of β, the variation of G with temperature was investigated. It can be seen in figure 8 that for iron, β and G^{-9} have about the same temperature dependence. Figure 6 shows, however, that β is proportional to G^{-3}. Therefore, it can be concluded that $F(T)$ contains a temperature-dependent factor in addition to G.

EVALUATION OF RESULTS

The statement is made in reference 9 (p. 344) that "a full interpretation of creep results in terms of dislocation theory would be welcome but has not yet been achieved." In the present report two contributions are made to a fuller understanding of creep in terms of both rate-process theory and dislocation theory. The first contribution is the quantitative dependence of creep on physical constants of the material; the second is an understanding of the origin of the large negative entropy of activation ΔS_a, which was a source of difficulty in previous treatments (references 3 and 13).

Although the previous theories have been successful in showing the dependence of creep on stress and temperature, none has been able to show the quantitative dependence of creep on physical constants of materials. From a practical point of view, equations that relate creep to stress and temperature have useful engineering application in predicting the creep behavior of a given material. These equations can yield little useful information, however, to the metallurgist who must find criteria to use in synthesizing alloys for better creep resistance. A knowledge of the dependence of creep rate on physical constants of material would yield such criteria and therefore have a practical as well as an academic interest.

The empirical equation that gives the dependence of steady-state rate of creep on stress σ and absolute temperature T can be taken as

$$\log u = \log\left(\frac{d_1}{L}\frac{kT}{h}\right) - \frac{A + BT}{2.3\ kT} + \frac{\beta\sigma}{2.3\ kT} \qquad (21a)$$

The theoretical treatment in this report has shown that the parameters A, B, and β depend on physical constants of the material in the following manner:

$$A = VG_0\ f^2x^2 \qquad (23)$$

$$B = P' - Vx^2f^2G_0\alpha = P' - A\alpha \qquad (24)$$

$$\beta = qVxf(1 - 2F(T))\ \Big) \qquad (25)$$

Equation (21a) shows that, in order to have a low rate of creep, A and B must be large and β must be small. Equation (24) indicates that the effect of any physical constant on A will be in the opposite direction to its effect on B. Inasmuch as α in equation (24) is of the order of 10^{-4} and P' has been shown to be approximately constant for all pure annealed materials, the physical constant will have a greater effect on A than on B up to a temperature of 10^{4o} K. The most important physical constant in equation (23) is the modulus of rigidity at absolute zero G_0 inasmuch as the other constants do not vary much in order of magnitude among different materials. Empirically β was also found to decrease with increasing G. Pure metals having high moduli of rigidity will therefore be creep resistant.

The relation between modulus of rigidity G and modulus of elasticity E is given by

$$G = \frac{E}{2(1 + \mu)}$$

Inasmuch as values of μ usually run from about 0.3 to 0.4, the effect of variations in this factor for different materials will be so small that in general, pure metals will be creep resistant if their modulus of elasticity is high.

The extension of the theory to alloys, in particular heat-resisting alloys, will require modification to include the effects of strains produced by solid soluble atoms and precipitated particles. In view of the need for criteria that will hasten the development of alloys having heat-resisting properties, any indications that can be obtained from the present theory should be considered. The theory suggests the use of materials of high moduli of rigidity, and therefore of high moduli of elasticity, such as tungsten, molybdenum, and cemented tungsten carbide as matrix materials for heat-resisting alloys. An investigation conducted under the auspices of the National Defense Research Committee has shown that chrome-base alloys having high tungsten or molybdenum content generally have better heat-resisting properties than currently used alloys. The use of still higher percentages of these elements therefore merits serious consideration.

Aircraft Engine Research Laboratory,
    National Advisory Committee for Aeronautics,
        Cleveland, Ohio, January 18, 1946.

REFERENCES

1. (a) Seitz, Frederick: Creep and Secondary Plastic Effects. The
       Physics of Metals, ch. IX, sec. 32, McGraw-Hill Book Co.,
       Inc., 1943, pp. 133-141.

   (b) Seitz, Frederick: The Slip Process. The Physics of Metals,
       ch. VI, sec. 18, pp. 73-78.

2. Kanter, J. J.: The Problem of the Temperature Coefficient of
   Tensile Creep Rate. Trans. Am. Inst. Mining and Metallurgical
   Eng., vol. 131, 1938, pp. 385-404; discussion, pp. 405-418.

3. Kauzmann, Walter: Flow of Solid Metals from the Standpoint of
   the Chemical-rate Theory. Trans. Am. Inst. Mining and Metal-
   lurgical Eng., vol. 143, 1941, pp. 57-81.

4. Seitz, Frederick, and Read, T. A.: Theory of the Plastic Prop-
   erties of Solids. I: Jour. Appl. Phys., vol. 12, no. 2,
   Feb. 1941, pp. 100-118; II: vol. 12, no. 3, March 1941,
   pp. 170-186; III: vol. 12 no. 6, June 1941, pp. 470-486;
   IV: vol. 12, no. 7, July 1941, pp. 538-554.

5. Glasstone, Samuel, Laidler, Keith J., and Eyring, Henry: The
   Theory of Rate Processes. Ch. I, IV, and IX. McGraw-Hill
   Book Co., Inc., 1941, pp. 1-27, 153-201, and 477-551.

6. Taylor, G. I.: The Mechanism of Plastic Deformation of Crystals.
   Part I. Theoretical. Part II. Comparison with Observations.
   Proc. Roy. Soc. (London), ser, A, vol. 145, no. A 855, July 2,
   1934, pp. 362-402.

7. Orowan, E.: Zur Kristallplastizität. III, Über den Mechanismus
   des Gleitvorganges. Zeitschr. f. Phys., Bd. 89, Heft 9-10,
   Juni 26, 1934, pp. 634-659.

8. Polanyi, M.: Über eine Art Gitterstörung, die linen Kristall
   plastisch machen konnte. Zeitschr. f. Phys., Bd. 89, Heft 9-10,
   Juni 26, 1934, pp. 660-664.

9. Barrett, Charles S.: Structure of Metals. Ch. XVI. McGraw-
   Hill Book Co., Inc., 1943.

10. Parker, E. R., and Riisness, C. F.: Effect of Grain Size and
    Bar Diameter on Creep Rate of Copper at 200° C. Tech. Pub.
    No. 1690, Metals Tech., Am. Inst. Mining and Metallurgical
    Eng., vol. 11, Feb. 1944, pp. 1-8.

11. Kochendörfer, Albert: Theorie der Kristallplastizität.
    Zeitschr. f. Phys., Bd. 108, Heft 3-4, Jan. 26, 1938,
    pp. 244-264.

12. McVetty, P. G.: Creep of Metals at Elevated Temperatures -
    the Hyperbolic-Sine Relation Between Stress and Creep Rate.
    A.S.M.E. Trans., vol. 65, no. 7, Oct. 1943, pp. 761-769.

13. Dushman, Saul, Dunbar, L. W., and Huthsteiner, H.: Creep of
    Metals. Jour. Appl. Phys., vol. 15, no. 2, Feb. 1944,
    pp. 108-124.

14. Davis, Evan A.: Creep and Relaxation of Oxygen-Free Copper.
    Jour. Appl. Mech., vol. 10, no. 2, June 1943, pp. A101-A105.

15. Burghoff, H. L., and Blank, A. I.: Creep Characteristics of
    a Phosphorized Copper. Trans. Am. Inst. Mining and Metal-
    lurgical Eng., vol. 161, Feb. 1946, pp. 420-440.

16. Michel, André, and Cournot, Jean: Contribution à l'Étude de
    la Viscosité à Chaud. Application à Divers Métaux et Alliages.
    Congrès International pour l'Essai des Matériaux (Amsterdam),
    t. I, Sept. 12-17, 1927, pp. 397-430.

17. Anon.: Wrought 0.10 to 0.20 Per Cent Carbon Steels. Compila-
    tion of Available High-Temperature Creep Characteristics of
    Metals and Alloys, sec. A, Creep Data Sec., Joint Res.
    Committee on Effect of Temperature on the Properties of Metals,
    A.S.T.M., A.S.M.E., March 1938, p. 22.

18. Peirce, W. M., and Anderson, E. A.: Some Practical Aspects of
    Creep in Zinc. Trans. Am. Inst. Mining and Metallurgical Eng.,
    vol. 83, 1929, pp. 560-572.

19. Tyte, L. C.: The Rate of Viscous Flow of Metals. Part 1, Tin.
    Proc. Phys. Soc., vol. 50, part 2, no. 278, March 1, 1938,
    pp. 153-175.

20. Tyte, L. C.: The Rate of Viscous Flow of Metals. Part 2, Lead.
    Proc. Phys. Soc., vol. 51, part 2, no. 284, March 1, 1939,
    pp. 203-221.

21. Anon.: Handbook of Chemistry and Physics. Charles D. Hodgman,
    ed., Chem. Rubber Pub. Co. (Cleveland), 28th ed., 1944, p. 1643.

22. Hoyt, Samuel L.: Metals and Alloys Data Book. Reinhold Pub.
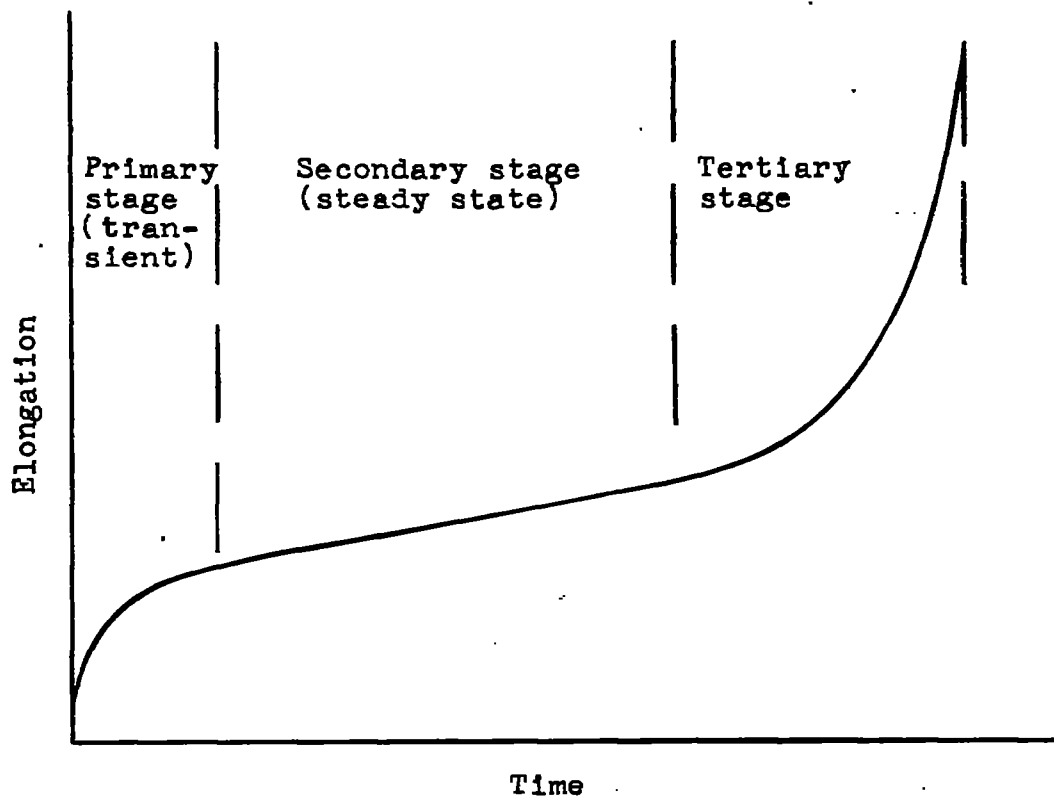   Corp., 1943, p. 100.

Figure 1.- Typical creep curve showing three stages.
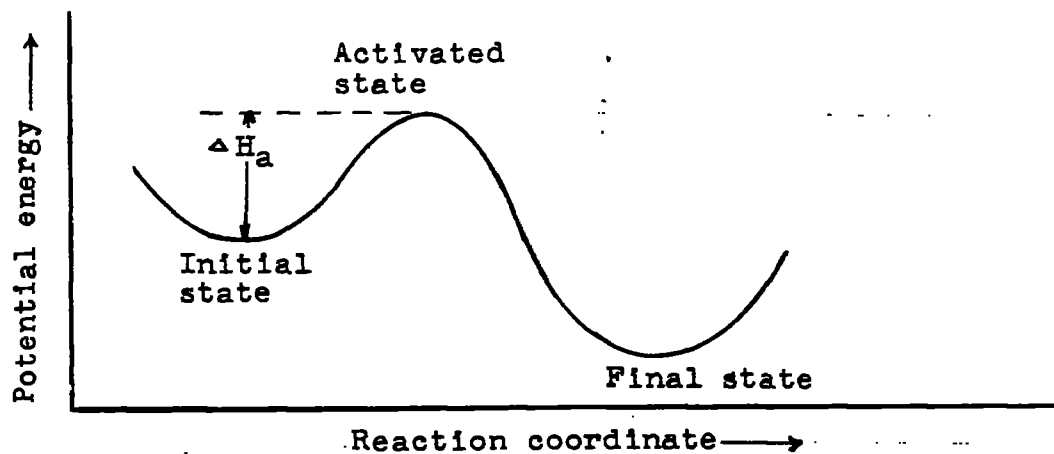
NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

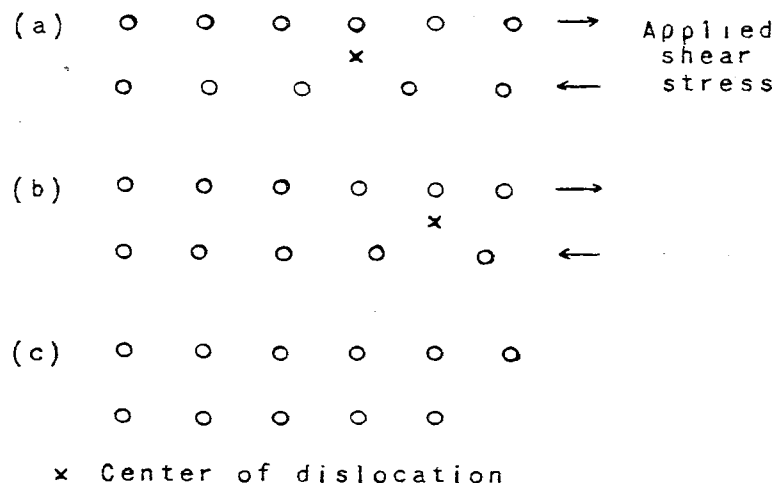
Figure 2.- Schematic representation of potential barrier.

(a)   o   o   o   o   o   o   →   Applied
                      x   shear
       o   o   o   o   o   ←   stress

(b)   o   o   o   o   o   o   →
                      x
       o   o   o   o   o   ←

(c)   o   o   o   o   o   o

       o   o   o   o   o

x Center of dislocation

Figure 3. - Schematic representation of displacement of
   two planes by motion of a dislocation.

a   b   c   d

e   f   g   h
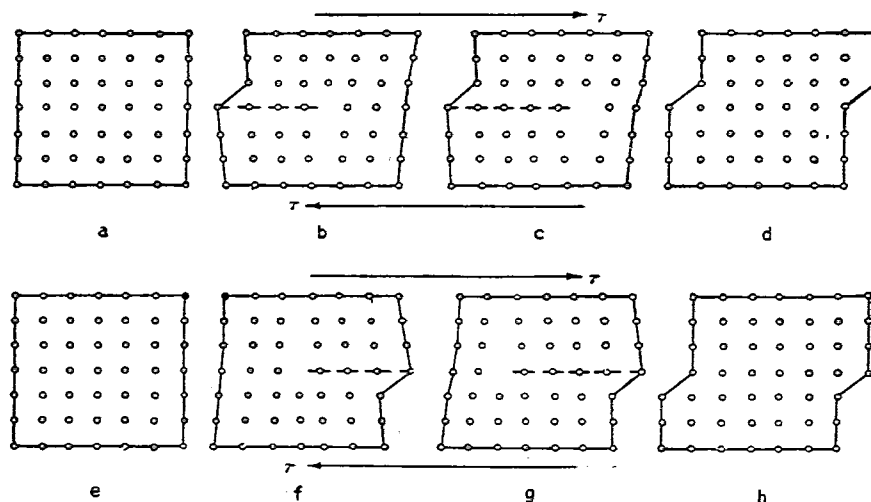
Figure 4. - Generation and movement of a dislocation.   In
   sketches a, b, c, and d a positive dislocation moves to
   the right; in e, f, g, and h a negative one moves to the
   left; the resulting deformation is identical. (Taylor.)
   (Reproduced from fig. 1 of reference 9, p. 335.)
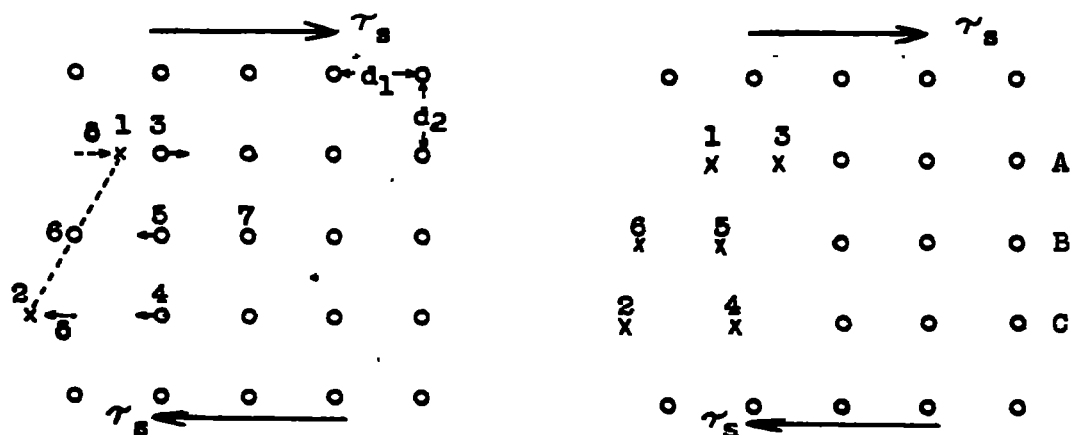
x Atoms that have made critical oscillations
o Atoms that are still unaffected by the oscillations

(a) Activated complex for a
positive dislocation.

(b) Dislocation formed by
rearrangement of the activated
complex.

Potential contribution
of row A

Potential contribution
of row C

Total potential field
in row B
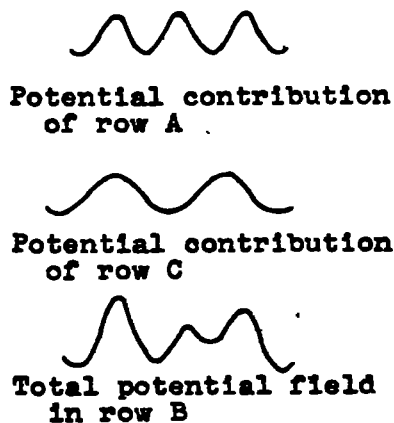
(c) Determination of potential
field in row B.
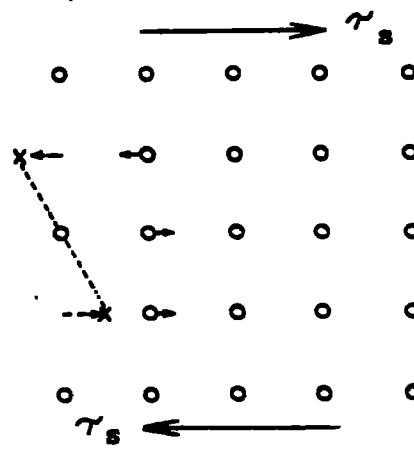
(d) Activated complex for a
negative dislocation

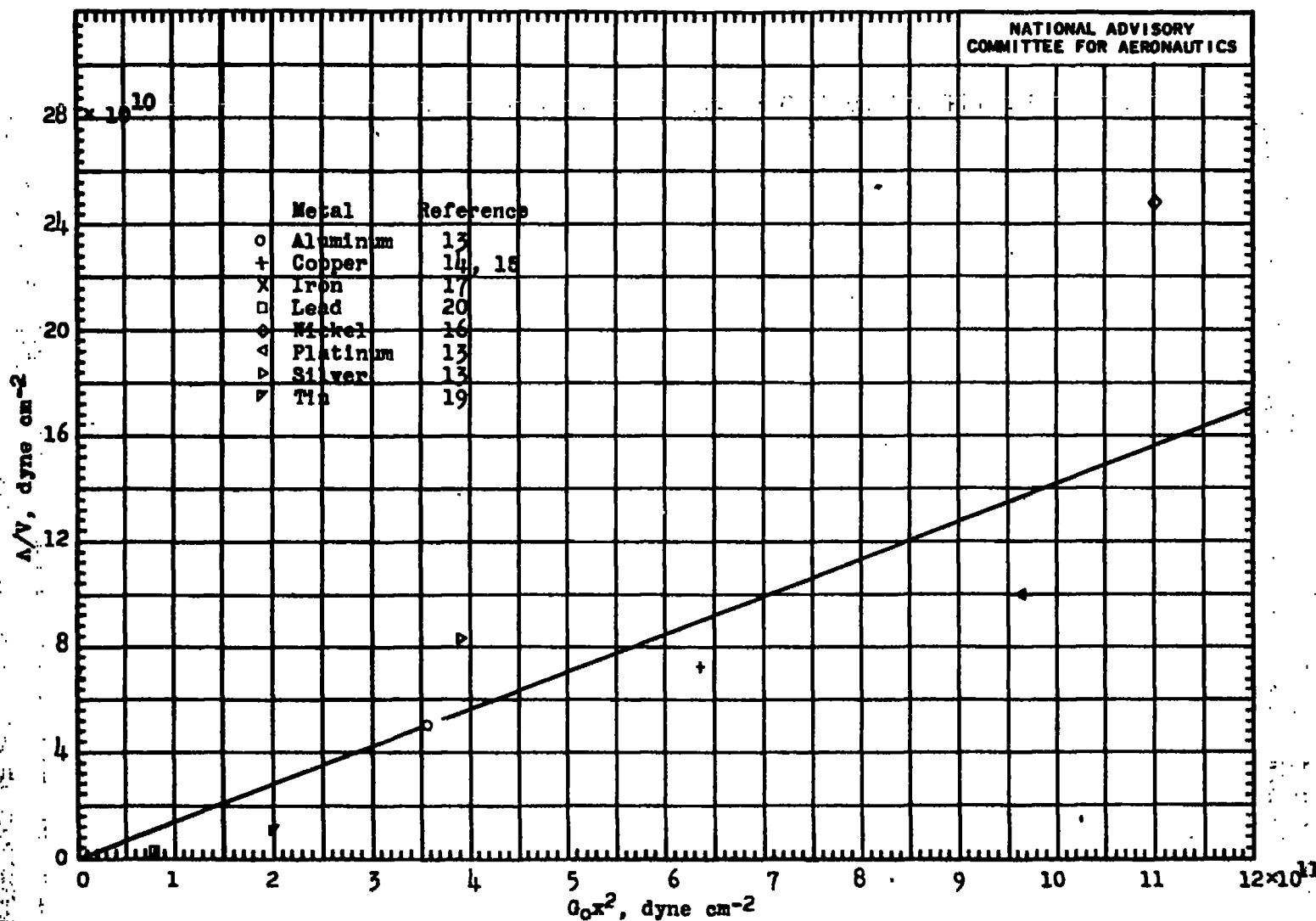Figure 5.- Generation of a dislocation at a positive source.

NATIONAL ADVISORY
COMMITTEE FOR AERONAUTICS

Fig. 6

Metal	Reference
o Aluminum	13
+ Copper	14, 15
X Iron	17
□ Lead	20
◇ Nickel	16
◁ Platinum	13
▷ Silver	13
▽ Tin	19

Figure 6. - Comparison of experimental data for factor A of equation (21) with theory.

Fig. 7
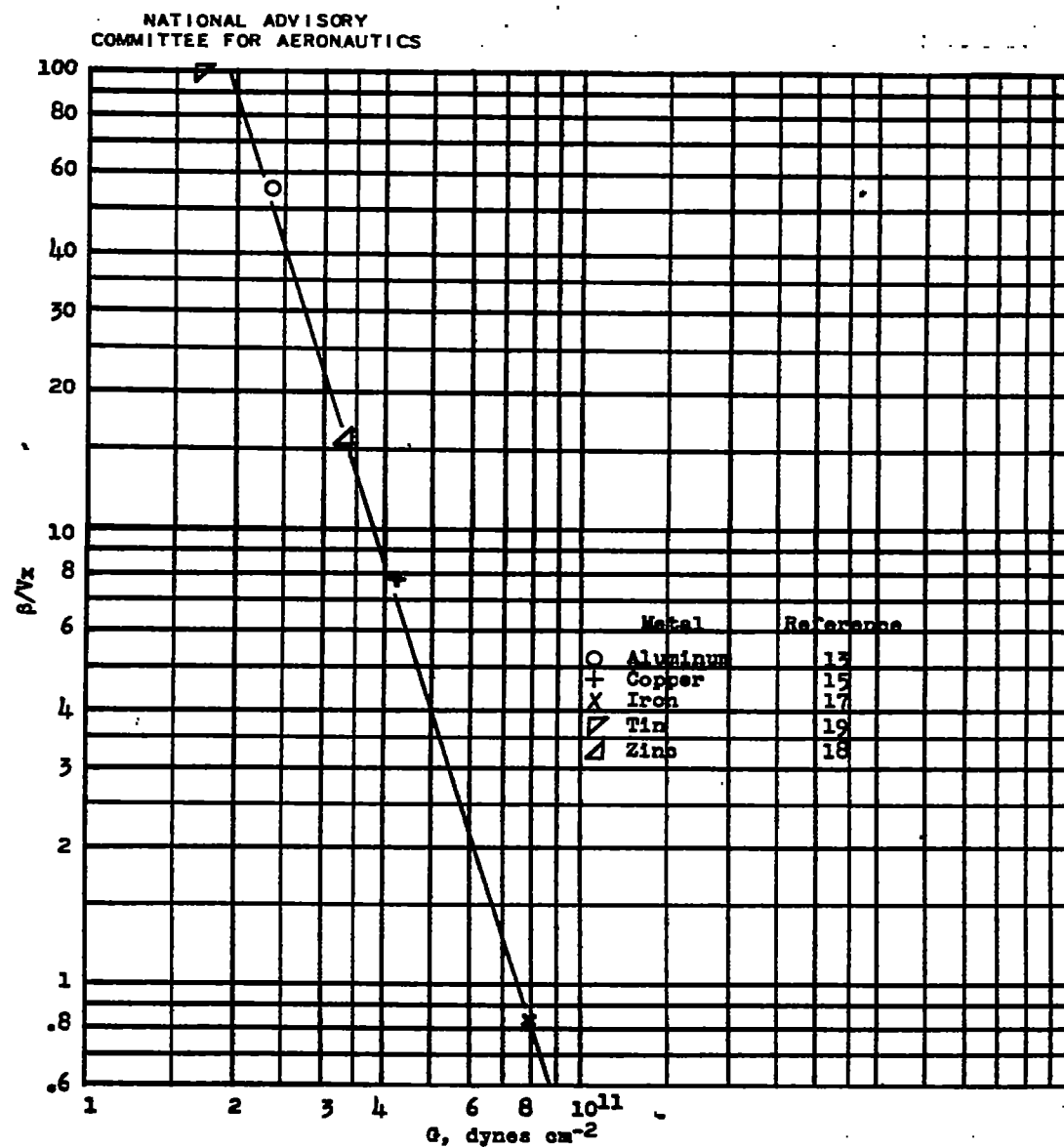
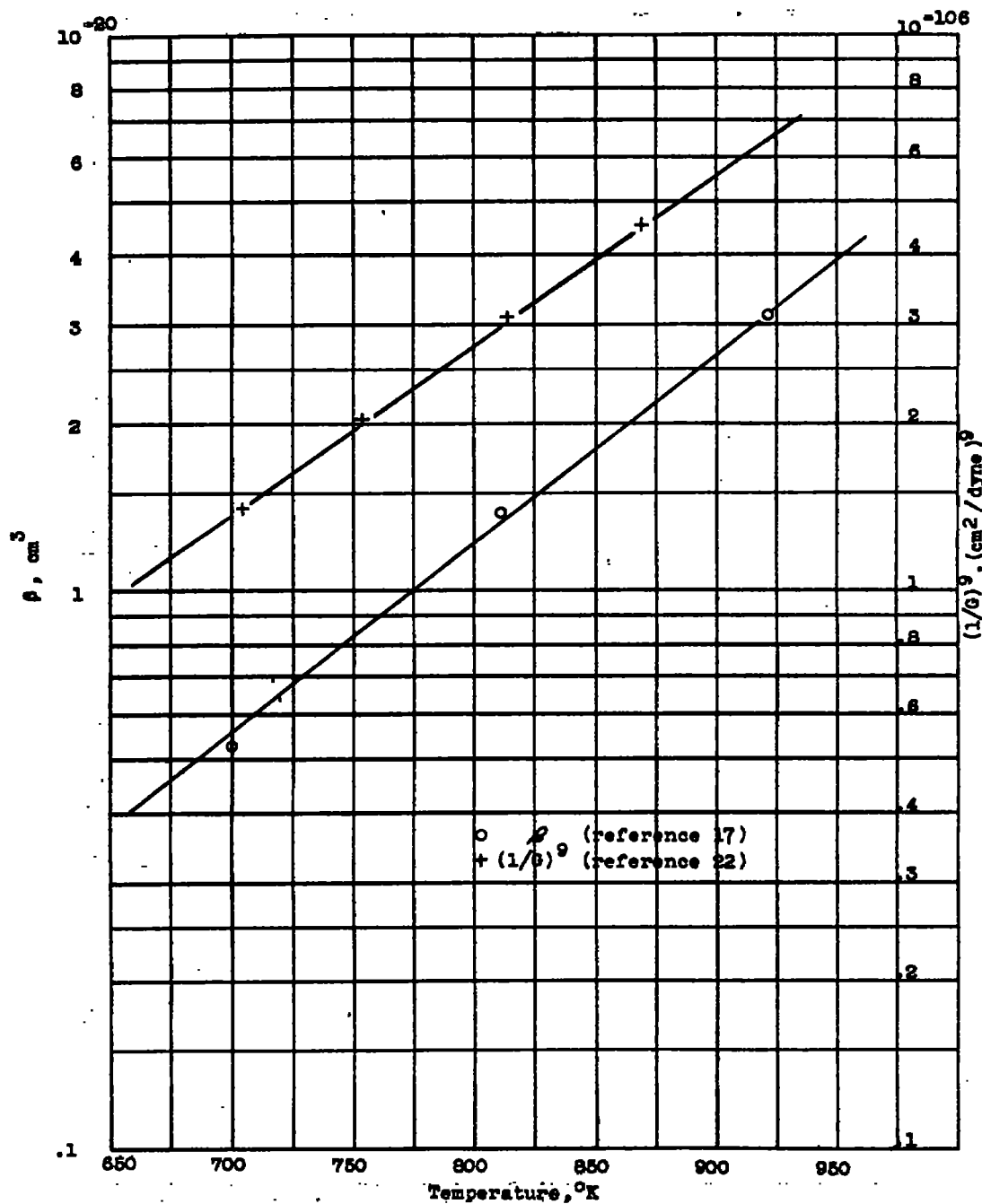Figure 7. - Room temperature dependence of β/Vx on the modulus of rigidity G.

Figure 8.- Temperature dependence of β and 1/G for iron.